

Time-Varying Cosmological Term: Emergence and Fate of a FRW Universe

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A time-varying cosmological “constant” Λ is consistent with Einstein’s equation, provided matter and/or radiation is created or destroyed to compensate for it. Supposing an empty primordial universe endowed with a very large cosmological term, matter will emerge gradually as Λ decays. Provided only radiation or ultrarelativistic matter is initially created, the universe starts in a nearly de Sitter phase, which evolves towards a FRW régime as expansion proceeds. If, at some cosmological time, the cosmological term begins increasing again, as presently observed, expansion will accelerate and matter and/or radiation will be transformed back into dark energy. It is shown that such accelerated expansion is a route towards a new kind of gravitational singular state, characterized by an empty, conformally transitive spacetime in which all energy is dark.

1. INTRODUCTION

One of the fundamental problems of cosmology is whether the universe will someday re-collapse in a big crunch, or will expand forever becoming increasingly cold and empty. Recent cosmological observations involving both supernovae [1] and the cosmic microwave background [2] suggest that the universe expansion is accelerating, an effect which points to the presence of some kind of negative-pressure energy, generically called dark energy. This energy is usually described by an equation of state of the form $\omega = p/\rho$, where ω is a parameter, not necessarily constant, and p and ρ are respectively the dark energy pressure and density. Cosmic acceleration requires that $\omega < -1/3$. The simplest explanation for dark energy is a cosmological term Λ , for which $\omega = -1$. Other popular, though somewhat bizarre possibilities are quintessence [3], a cosmic scalar field in which $-1 < \omega < -1/3$, and phantom energy [4], scalar field models presenting a quite unusual kinetic term, for which $\omega < -1$. Depending on both the model and the value of the parameters, different fates for the universe can be achieved, which range from a simple re-collapse, passing through a bleak eternal expansion, to an astonishing big rip end [5].

Working in the context of a *dynamical cosmological term*, a different and unexplored possibility for the fate of the universe will be presented. We begin in section 2 where, for the sake of completeness, we show that a dynamical cosmological term is consistent with general relativity, provided matter and/or radiation is created to make overall energy conserved. In section 3, the Friedmann equations for a time-dependent Λ are obtained, and in section 4 a qualitative analysis of such evolution equations is made for the specific case of a *time-decaying* Λ . In

spite of the lacking of an adequate understanding of the physics associated with a cosmological term, in particular of the law governing its time evolution, it is pointed out that an appropriate Λ -evolution could explain most of the present-day observations, without necessity of any exotic machinery. In order to comply with the present observational data, we consider in section 5 the case of an increasing Λ , which will produce an accelerated universe expansion. It is then pointed out that this accelerated expansion corresponds to a new route to a collapsing universe, whose final stage is an empty, singular, conformally transitive spacetime in which all energy is in the form of dark energy [6].

2. DYNAMIC DARK ENERGY AND EINSTEIN’S EQUATION

In the presence of a cosmological constant Λ , Einstein’s equation assumes the form [7]

$$G^\mu{}_\nu \equiv R^\mu{}_\nu - \frac{1}{2} \delta^\mu{}_\nu R = \frac{8\pi G}{c^4} \left[T^\mu{}_\nu + \frac{c^4 \Lambda}{8\pi G} \delta^\mu{}_\nu \right], \quad (1)$$

where $T^\mu{}_\nu$ is the energy-momentum density tensor of the source field. Put together, the Bianchi identity

$$\nabla_\mu G^\mu{}_\nu = 0 \quad (2)$$

and the source “covariant conservation law”

$$\nabla_\mu T^\mu{}_\nu = 0 \quad (3)$$

imply that the cosmological constant cannot present any kind of space or time dependence. In other words, it must be a true constant. On the other hand, since inflationary models require a very high Λ at the early stages of the universe [8] and present-day observations indicate a much smaller value [1, 2], that constancy restriction appears as one of the central problems of cosmology [9].

Of course, the vanishing of a (gravitational field dependent) covariant divergence as in (3) is no true conservation law: it yields no time-conserved “charge”. The

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role of such a “covariant conservation” is to regulate the exchange of energy and momentum between the source fields (generically called matter, from now on) and the gravitational field. Furthermore, it is not necessarily true in all circumstances. It would not be expected to hold, for example, if matter (plus the gravitational field it gives rise to) is being *created* from an independent source. As already pointed out in the literature [10], a time-decaying cosmological term can be such a source.

We observe to begin with that, in the presence of a non-constant cosmological term, what is imposed by the use of the Bianchi identity (2) in Einstein’s equation (1) is, instead of (3), the condition

$$\nabla_\mu [T^\mu{}_\nu + \Lambda^\mu{}_\nu] = 0, \quad (4)$$

where $\Lambda^\mu{}_\nu = \varepsilon_\Lambda \delta^\mu{}_\nu$ is the dark energy-momentum tensor associated with the cosmological term, with

$$\varepsilon_\Lambda = \frac{c^4 \Lambda}{8\pi G} \quad (5)$$

the corresponding energy density. The energy-momentum of matter alone is consequently not covariantly conserved. Only its sum with the dark energy-momentum tensor is. The covariant conservation law (4) can be interpreted as a constraint regulating the exchange of energy and momentum between matter, gravitation and the cosmological term. In other words, it says how the cosmological dark energy can be transformed into ordinary matter plus the gravitational field it engenders, or vice-versa. Assuming that Λ depends only on the cosmological time t [11], the covariant conservation (4) is equivalent to (we use $i, j, k = 1, 2, 3$ to denote space indices)

$$\nabla_\mu T^\mu{}_i = 0, \quad (6)$$

and

$$\nabla_\mu T^\mu{}_0 = -\frac{c^3}{8\pi G} \frac{d\Lambda}{dt}. \quad (7)$$

We see now from Eq. (7) that a time-decaying Λ implies that the source energy-momentum tensor is not *covariantly* conserved, and consequently matter must necessarily be created as the cosmological term decays. Notice that the total energy of the universe is conserved despite matter creation. To see that, it is enough to take Einstein’s equation with a cosmological term in the so called potential form [12],

$$\partial_\rho (\sqrt{-g} S^{\rho\mu}{}_\nu) = \frac{8\pi G}{c^4} [\sqrt{-g} (t^\mu{}_\nu + T^\mu{}_\nu + \Lambda^\mu{}_\nu)], \quad (8)$$

where $S^{\rho\mu}{}_\nu = -S^{\mu\rho}{}_\nu$ is the superpotential, and $t^\mu{}_\nu$ is the energy-momentum pseudotensor of the gravitational field. Due to the anti-symmetry of the superpotential in the first two indices, the total energy-momentum density, which includes the gravitational, the matter and the cosmological parts, is conserved:

$$\partial_\mu [\sqrt{-g} (t^\mu{}_\nu + T^\mu{}_\nu + \Lambda^\mu{}_\nu)] = 0. \quad (9)$$

This is actually the Noether conservation law obtained from the invariance of the theory under a general transformation of the spacetime coordinates.

It is important to remark that the covariant conservation law (7) is different from that appearing in quintessence models [3]. In fact, in such models the energy-momentum tensor of the scalar field that replaces the cosmological term is itself covariantly conserved, and consequently there is no matter creation. On the other hand, despite the presence of continuous matter creation, this mechanism is different also from the C -field theory of Hoyle and Narlikar [13] as in the present model no scalar field is introduced, but only a cosmological term whose time decaying turns out to be linked to the matter energy density evolution through the Friedmann equations.

3. FRIEDMANN EQUATIONS

The starting point of our considerations will be an empty universe endowed with a very large—possibly infinite [14]—decaying cosmological term. In the extreme case of an infinite Λ , this spacetime is given by a singular cone-space, transitive under proper conformal transformations [6]. If we assume that the ensuing newly created matter is a homogeneous and isotropic fluid, it is natural to consider that the metric tensor of this gravitational field be of the Friedmann-Robertson-Walker (FRW) type

$$ds^2 = c^2 dt^2 - a^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where $a = a(t)$ is the expansion factor and k is the curvature parameter of the space section. The coordinates are taken for a comoving observer in relation to the lines of flux of a perfect fluid, whose energy-momentum tensor has the form

$$T^\mu{}_\nu = (\varepsilon_m + p_m) u^\mu u_\nu - p_m \delta^\mu{}_\nu, \quad (10)$$

with p_m and ε_m the pressure and the energy density of the created matter. Denoting $x^0 = ct$, $x^1 = r$, $x^2 = \theta$, and $x^3 = \phi$, the non-zero energy-momentum components for that homogeneous, isotropic fluid will be:

$$T^1{}_1 = T^2{}_2 = T^3{}_3 = -p_m,$$

$$T^0{}_0 = \varepsilon_m.$$

The conservation law (7) in this case becomes

$$\frac{d\varepsilon_m}{dt} + 3H(\varepsilon_m + p_m) = -\frac{d\varepsilon_\Lambda}{dt}, \quad (11)$$

with

$$H = \frac{1}{a} \frac{da}{dt}$$

the Hubble parameter. This is actually one of the Friedmann equations. In fact, it can be seen that it follows from the combination of the usual Friedmann equations

$$\left(\frac{da}{dt}\right)^2 = \left[\frac{8\pi G}{3c^2}\varepsilon_m + \frac{\Lambda c^2}{3}\right]a^2 - kc^2 \quad (12)$$

and

$$\frac{d^2a}{dt^2} = \left[\frac{\Lambda c^2}{3} - \frac{4\pi G}{3c^2}(\varepsilon_m + 3p_m)\right]a, \quad (13)$$

provided Λ is time dependent. It is important to remark that, according to this model, matter is not created at once in a big bang. It emerges as long as the cosmological term decays, in a gradual process.

Let us now suppose that the newly created ordinary matter satisfies an equation of state of the form

$$p_m = \omega_m \varepsilon_m, \quad (14)$$

with $0 \leq \omega_m \leq 1$ a parameter that depends on the specific kind of matter. Of course, the matter content of the universe can be made up of more than one component, each one satisfying an equation of state with a different ω_m . These possibilities should be taken into account in a comprehensive description of the universe evolution. Here, however, it is enough for our purposes to consider a one-component. In this case, Eq. (11) becomes

$$\frac{d\varepsilon_m}{dt} + 3H(1 + \omega_m)\varepsilon_m = -\frac{d\varepsilon_\Lambda}{dt}. \quad (15)$$

The second Friedmann equation, on the other hand, can be written in the form

$$\frac{d^2a}{dt^2} = \frac{8\pi G}{3c^2} \left[\varepsilon_\Lambda - \frac{1}{2}(1 + 3\omega_m)\varepsilon_m \right] a. \quad (16)$$

4. EVOLUTION ANALYSIS

A. General Case

The Friedmann equation (15) establishes a connection between the evolutions of ε_m and ε_Λ . In fact, if ε_m depends on time through the expansion factor a , the dark energy density ε_Λ will also have the same dependence on a , and vice-versa. In principle any behavior is possible for ε_m and ε_Λ , although it is usual to suppose that ε_m evolves as a power law in the expansion factor,

$$\varepsilon_m = \alpha a^{-n}, \quad (17)$$

with α a constant and n a number (integer or not) [15]. In this case, Eq. (15) implies

$$\varepsilon_\Lambda = \frac{3(1 + \omega_m) - n}{n} \varepsilon_m, \quad (18)$$

where we have assumed a vanishing integration constant. In the presence of a dynamical cosmological term, therefore, depending on the parameters n and ω_m , the energy densities ε_m and ε_Λ may eventually be of the same order, as strongly suggested by present observations [16]. Of course, these parameters can also lead to periods in which ε_m and ε_Λ are completely different. It is interesting to observe that the case $n = 0$, which would correspond to an equilibrium between matter creation and universe expansion ($\varepsilon_m = \text{constant}$), is excluded by the Friedmann equations. Notice furthermore that, for ε_Λ constant, Eq. (15) yields the solution $\varepsilon_m \sim a^{-3(1+\omega_m)}$. For a time-decaying ε_Λ , however, n is required to be in the interval

$$0 < n < 3(1 + \omega_m). \quad (19)$$

Since matter is continuously created, it is natural that ε_m evolves at a rate slower than $a^{-3(1+\omega_m)}$, which would be its behavior if matter were not being created.

On the other hand, using the equation of state (14), as well as the relations (17) and (18), the Friedmann equation (16) becomes

$$\frac{d^2a}{dt^2} = \frac{3(1 + \omega_m)\beta^2}{2} \left(\frac{2 - n}{n}\right) a^{1-n}, \quad (20)$$

where $\beta^2 = 8\pi G\alpha/3c^2$. We see from this equation that, for $n = 2$, $n > 2$ and $n < 2$, the universe expansion acceleration will be respectively zero, negative and positive. This property could eventually explain why the acceleration was negative in the past and positive today, as suggested by recent observational data [16]. Furthermore, in the case of a positive acceleration ($n < 2$), the ranges $n > 1$ and $n < 1$ will represent respectively the cases in which the acceleration is decreasing or increasing, with the value $n = 1$ representing a universe with a constant expansion acceleration, given by

$$\frac{d^2a}{dt^2} = \frac{3(1 + \omega_m)\beta^2}{2}.$$

In this case, $a \sim t^2$, and we have the relations

$$\Lambda \sim a^{-1} \sim H^2 \sim t^{-2}.$$

We notice finally that, as the parameters ω_m and n have very limited ranges, the above results do not change very much when ω_m is assumed to vary slowly with the cosmological time, or the matter content of the universe has more than one component.

B. The Flat Case

Recent observational data favor a universe with flat spatial sections ($k = 0$). In this case, it is possible to find an explicit time-dependence for the cosmological term, which is valid for any value of the parameters n and ω_m .

In fact, for $k = 0$ the Friedmann equation (12) can be written in the form

$$\left(\frac{da}{dt}\right)^2 = \frac{3\beta^2(1+\omega_m)}{n} a^{2-n}, \quad (21)$$

or equivalently

$$a^{n/2-1} da = \left(\frac{3(1+\omega_m)\beta^2}{n}\right)^{1/2} dt. \quad (22)$$

Assuming a vanishing integration constant [17], the solution is found to be

$$a = \left(\frac{3n(1+\omega_m)\beta^2}{4}\right)^{2/n} t^{2/n}. \quad (23)$$

As a consequence, the matter and the dark energy densities will present the behavior [18]:

$$\varepsilon_m \sim \varepsilon_\Lambda \sim t^{-2}. \quad (24)$$

Due to relation (5), and using Einstein's equation, we have also

$$\Lambda \sim R \sim t^{-2}. \quad (25)$$

This is the time-dependence of Λ and R for any value of the parameters n and ω_m . We see from this behavior that both the cosmological term Λ and the scalar curvature R diverge at the initial time, which signals the existence of an initial singularity.

As an example related to the initial period of the universe, let us assume what can be called a *flat lux* hypothesis, according to which the newly created matter satisfies the ultra-relativistic equation of state [19]

$$\varepsilon_m = 3p_m, \quad (26)$$

which corresponds to $\omega_m = 1/3$. In this case, the trace of Einstein's equation (1) gives [20]

$$R = -4\Lambda, \quad (27)$$

where we have used $T \equiv T^\mu{}_\mu = \varepsilon_m - 3p_m = 0$. As long as only radiation and ultrarelativistic matter is created, the scalar curvature is completely determined by Λ , and in this sense the universe can be considered to be in a nearly de Sitter phase (of course, since the cosmological term is not constant, it is not a de Sitter spacetime in the ordinary sense). Now, for a positive cosmological term ($\Lambda > 0$), the scalar curvature is given by $R = -12/L^2$, where L is the de Sitter length-parameter, or de Sitter "radius". In terms of L , the dark energy density (5) becomes

$$\varepsilon_\Lambda = \frac{3c^4}{8\pi GL^2}. \quad (28)$$

On the other hand, the Friedmann equation (15) assumes the form

$$\frac{d\varepsilon_m}{dt} + 4H\varepsilon_m = -\frac{d\varepsilon_\Lambda}{dt}. \quad (29)$$

For ε_Λ constant, it yields the usual solution $\varepsilon_m \sim a^{-4}$. For a time-decaying ε_Λ , however, we get the relation

$$\varepsilon_\Lambda = \frac{4-n}{n} \varepsilon_m, \quad (30)$$

where now $0 < n < 4$. Equations (28) and (30) imply that, as long as matter is ultrarelativistic, the de Sitter radius expands according to

$$L^2 = \frac{c^2}{3\beta^2} \left(\frac{n}{4-n}\right) a^n. \quad (31)$$

5. FINAL REMARKS

In the context of a Λ cosmology, we can say that the dynamics of the universe was dominated by a very large positive cosmological term during the period of primordial inflation, or even by the eventual extreme possibility of an infinite Λ [14]. Quantum fluctuations could then give rise to a de Sitter spacetime, which is well known to exhibit a horizon [21] at the de Sitter length $L = \sqrt{3/\Lambda}$. If, at the Planck time, L is assumed to coincide with the Planck length l_P , the cosmological "constant" would, at that moment, have the value

$$\Lambda = 3/(l_P)^2 \simeq 1.2 \times 10^{66} \text{ cm}^{-2}.$$

The dark energy density, on the other hand, would be

$$\varepsilon_\Lambda \simeq 10^{112} \text{ erg/cm}^3.$$

At this time, therefore, most of the energy density of the universe would be in the dark energy form. We notice in passing that, since present-day observations indicate that

$$\varepsilon_\Lambda^0 \simeq 10^{-8} \text{ erg/cm}^3,$$

the evolving mechanism implied by the Friedmann equations with a time-decaying cosmological term could eventually give an account of this difference. As the Λ term decays and the universe expands, matter and/or radiation is gradually created, giving rise to a FRW universe. Despite the continuous creation of matter, however, the total energy density of the universe—which includes, in addition to the matter and the dark energy densities, the energy density of the evolving gravitational field they generate—is conserved. We remark once more that, according to this mechanism, matter is not created at once in a big bang, but gradually as the cosmological term decays.

Models with a time decaying cosmological term [22] have already been extensively considered in the literature [23]. The basic idea underlying these models is to try to explain how a large primordial Λ can present a small value today. All of them are essentially phenomenological in nature, and based preponderantly on dimensional

arguments. Here, however, instead of adopting a phenomenological point of view, we have followed a theoretical approach based almost exclusively on the equations governing the universe evolution, that is, on the Friedmann and on the matter equations of state. Since a new degree of freedom, connected with the time-evolving cosmological term, is introduced, there remains in the theory a free parameter—the cosmological term—whose time evolution has eventually to be determined by further fundamental physics [24]. Of course, to explain why ε_Λ and ε_m are approximately of the same order today, and eventually why the acceleration was negative in the past and positive today, as strongly suggested by recent observations, a quite specific time evolution for Λ is necessary. This question, however, remains as one of the mysteries involving the nature of dark energy, an open problem to be investigated. The important point is to observe that a dynamical cosmological term endowed with an appropriated time evolution contains enough free parameters to allow a wide range of scenarios for the cosmological evolution, including the main features favored by recent astronomical data, and does not require any further exotic ingredient (as, for example, phantom energy, quintessence models, or even modifications of the gravitational theory) to consistently describe the dynamical evolution of the universe [25].

An important point is to observe that, in order to allow the formation of the cosmological structures we see today (galaxies, clusters of galaxies, and so on), the universe necessarily has passed through a period of non-accelerating expansion, which means that the cosmological term must have assumed a tiny value during some cosmological period in the past. On the other hand, recent observations [1, 2] indicate that the universe is presently entering another exponential expansion era. Even though we still lack an adequate understanding of the basic physics associated with the evolution of the cosmological term, the above facts put together suggest a primordial universe characterized by a very large Λ , including eventually the possibility of an infinite Λ , followed by a somehow decaying cosmological term which, after keeping a minimum value during some period, has entered a new increasing period. If this is true, in the same way a decaying Λ implies that matter and/or radiation be created, conservation of energy requires that matter and/or radiation be transformed into dark energy by a time increasing Λ term [26].

Now, it is frequently argued that, if this new phase of exponential expansion is in fact occurring, the universe would be driven either to a bleak future, a state that could be called cosmic loneliness, or eventually to a complete disintegration which has been called “big rip” [5]. However, as far as a time increasing Λ -term implies that matter and/or radiation be transformed into dark energy, such a mechanism could eventually lead the universe to a state in which the whole energy would be in the form of dark energy. In other words, a time increasing Λ does not necessarily mean that the universe will

disperse and become colder, but that it may be moving towards a new kind of singular state. If led to the extreme situation of an infinite cosmological term, the universe would achieve a singular state characterized by an empty, causally disconnected, conformally transitive spacetime (a brief description of the basic geometrical properties of this spacetime is given in the Appendix). Of course, whether quantum effects will or not preclude such “collapse” is an open question.

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APPENDIX: THE INFINITE- Λ SPACETIME

A.1 Kinematic groups: transitivity

The kinematic group of any spacetime will always have a subgroup accounting for both the isotropy of space (rotation group) and the equivalence of inertial frames (boosts). The remaining transformations, which can be either commutative or not, are responsible for the homogeneity of space and time. This holds, of course, for usual Galilean and other conceivable non-relativistic kinematics [27], but also for special-relativistic kinematics. The best known relativistic example is the Poincaré group \mathcal{P} , naturally associated with the Minkowski spacetime M as its group of motions. It contains, in the form of a semi-direct product, the Lorentz group $\mathcal{L} = SO(3, 1)$ and the translation group \mathcal{T} . The latter acts transitively on M and its manifold is just M . Indeed, Minkowski spacetime is a homogeneous space under \mathcal{P} , actually the quotient $M \equiv \mathcal{T} = \mathcal{P}/\mathcal{L}$. The invariance of M under the transformations of \mathcal{P} reflects its uniformity. The Lorentz subgroup provides an isotropy around a given point of M , and translation invariance enforces this isotropy around any other point. This is the usual meaning of “uniformity”, in which \mathcal{T} is responsible for the equivalence of all points of spacetime.

A.2 The case of the de Sitter spacetime

The de Sitter $dS(4, 1)$ and anti-de Sitter $dS(3, 2)$ spacetimes are the only possible uniformly *curved* four-dimensional metric spacetimes [19]. They are maximally-symmetric, in the sense that they can lodge the maximum number of Killing vectors. These spacetimes are related respectively to a positive and to a negative cosmological term Λ , and their groups of motions are respectively the de Sitter $SO(4, 1)$ and anti-de Sitter

$SO(3, 2)$ groups. Both spaces are homogeneous [28]:

$$dS(4, 1) = SO(4, 1)/SO(3, 1),$$

$$dS(3, 2) = SO(3, 2)/SO(3, 1).$$

In addition, each group manifold is a bundle with the corresponding de Sitter or anti-de Sitter space as base space, and the Lorentz group \mathcal{L} as fiber [29].

Let us then analyze the kinematic group of the de Sitter spacetime ($\Lambda > 0$). In terms of the stereographic coordinates x^a ($a, b, \dots = 0, 1, 2, 3$), the generators of infinitesimal de Sitter transformations are written as [30]

$$J_{ab} = \eta_{ac} x^c P_b - \eta_{bc} x^c P_a \quad (32)$$

and

$$T_a = \left(L P_a - \frac{1}{4L} K_a \right), \quad (33)$$

where

$$P_a = \frac{\partial}{\partial x^a} \quad \text{and} \quad K_a = (2\eta_{ab} x^b x^c - \sigma^2 \delta_a^c) P_c \quad (34)$$

are, respectively, the generators of translations and *proper* conformal transformations. In the above expressions, $L = (3/\Lambda)^{1/2}$ is a length-parameter related to the curvature of the de Sitter space, and $\sigma^2 = \eta_{ab} x^a x^b$ is the Lorentz invariant interval, with $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$. The generators J_{ab} refer to the Lorentz subgroup \mathcal{L} , whereas T_a define the transitivity on the corresponding homogeneous space. According to Eq. (33), we see that the de Sitter spacetime is transitive under a mixture of translations and proper conformal transformations. The relative importance of each one of these transformations is determined by the value of the cosmological term.

A.3 Contraction limits

Let us begin by remarking that, on account of the quotient character of the de Sitter spacetime, geometry and algebra turns out to be deeply connected: any deformation in the algebras and groups will produce concomitant deformations in the imbedded spacetime. As an example, let us consider first the limit $\Lambda \rightarrow 0$ (which corresponds to $L \rightarrow \infty$). In this limit, as is well known [30],

the de Sitter group is contracted [31] to the Poincaré group $\mathcal{P} = \mathcal{L} \circ \mathcal{T}$. This group deformation will produce changes in the imbedded spacetime. In fact, the de Sitter spacetime reduces in this limit to the flat Minkowski space $M = \mathcal{P}/\mathcal{L}$, which is *transitive* under translations only.

In the limit $\Lambda \rightarrow \infty$ (which corresponds to $L \rightarrow 0$), the de Sitter group is contracted to the so called *second* or *conformal* Poincaré group \mathcal{Q} , the semi-direct product between Lorentz \mathcal{L} and the proper conformal group \mathcal{C} , that is, $\mathcal{Q} = \mathcal{L} \circ \mathcal{C}$ [32]. This group deformation will accordingly produce changes in the imbedded spacetime. In fact, in the limit of an infinite cosmological term, the de Sitter space is led to a four-dimensional cone-space [6], which we denote by N . Like Minkowski, the cone-space N is a homogeneous space, but under \mathcal{Q} : $N = \mathcal{Q}/\mathcal{L}$. The kinematical group \mathcal{Q} , as the Poincaré group, has the Lorentz group \mathcal{L} as the subgroup accounting for the isotropy of N . However, the proper conformal transformations introduce a new kind of homogeneity: instead of the ordinary translations, which defines the homogeneity on Minkowski spacetime, all points of N are equivalent through proper conformal transformations. In other words, the cone-space N is transitive under proper conformal transformations. On account of this conformal transitivity, the cone-space N can be said to be *conformally infinite*.

It is important to remark that the usual metric of the de Sitter spacetime becomes singular in the contraction process [6]. This is the reason why the ordinary notions of space distance and time interval fail to exist on N . However, the corresponding notions of *conformal space* and *conformal time* can be defined through the introduction of the conformal invariant metric [6]

$$\bar{\eta}_{ab} = \sigma^{-4} \eta_{ab}; \quad \bar{\eta}^{ab} = \sigma^4 \eta^{ab}. \quad (35)$$

As a direct inspection shows, $\bar{\eta}_{ab}$ is in fact invariant under the conformal Poincaré group \mathcal{Q} . Therefore, if $ds^2 = \eta_{ab} dx^a dx^b$ is the Minkowski interval, the corresponding cone-space “conformal interval” will be

$$d\bar{s}^2 = \bar{\eta}_{ab} dx^a dx^b. \quad (36)$$

It is worthy mentioning finally that, in this new (maximally-symmetric) spacetime physics will be quite unusual: the ordinary notions of space and time do not exist, there is no place for the usual concept of movement, no Planck length can be defined, and so on [33].

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